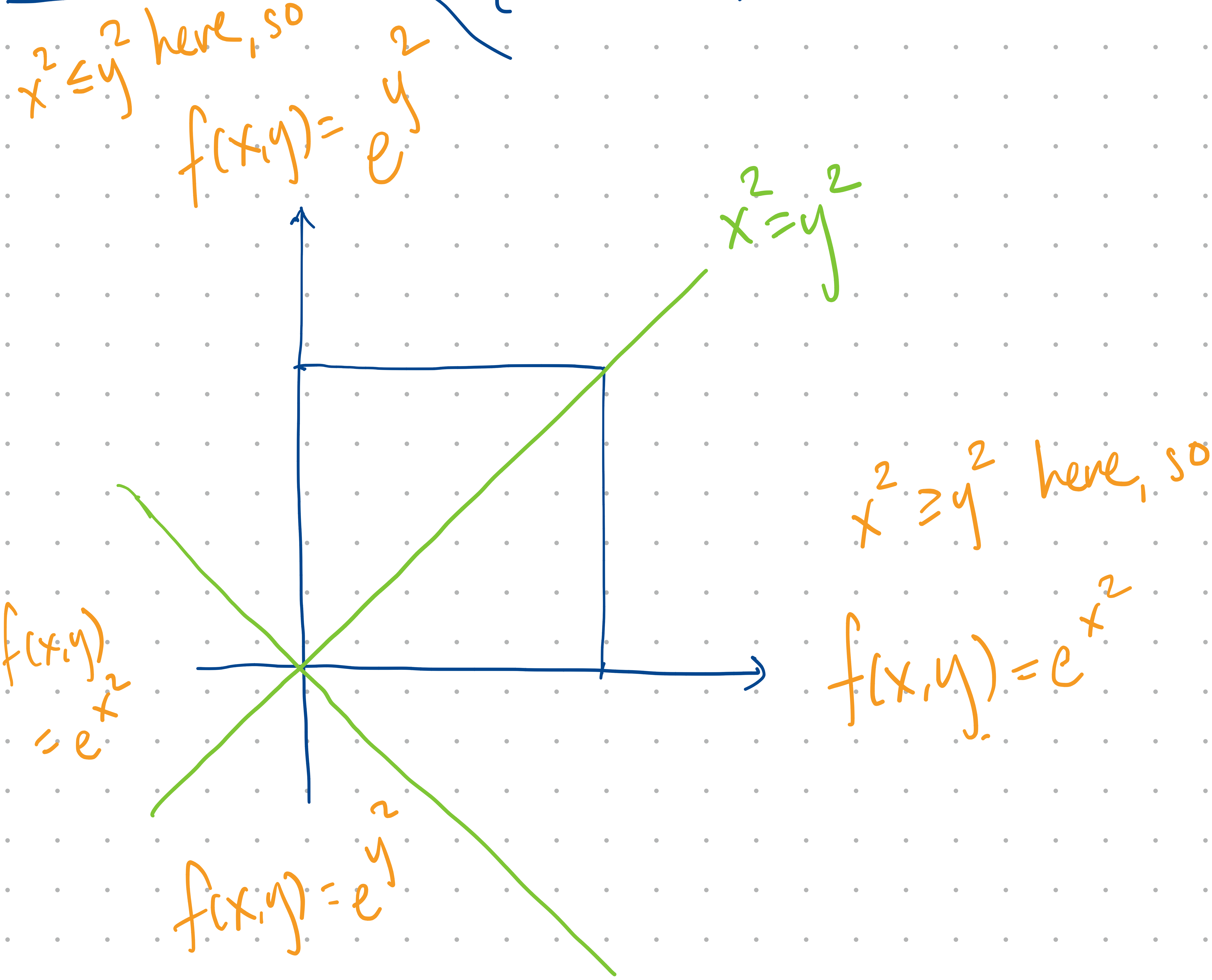
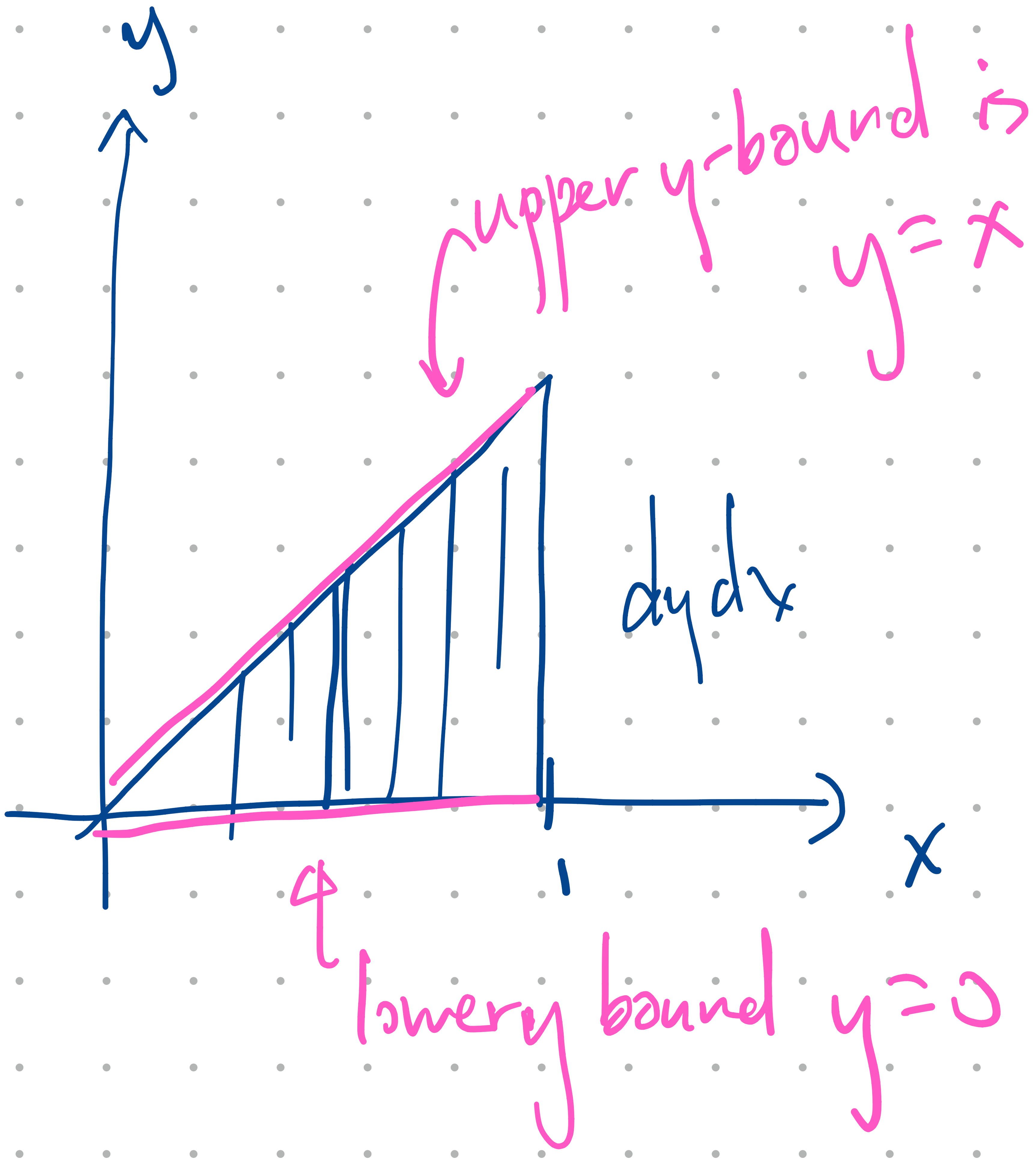
Stewart p.1105 #2)

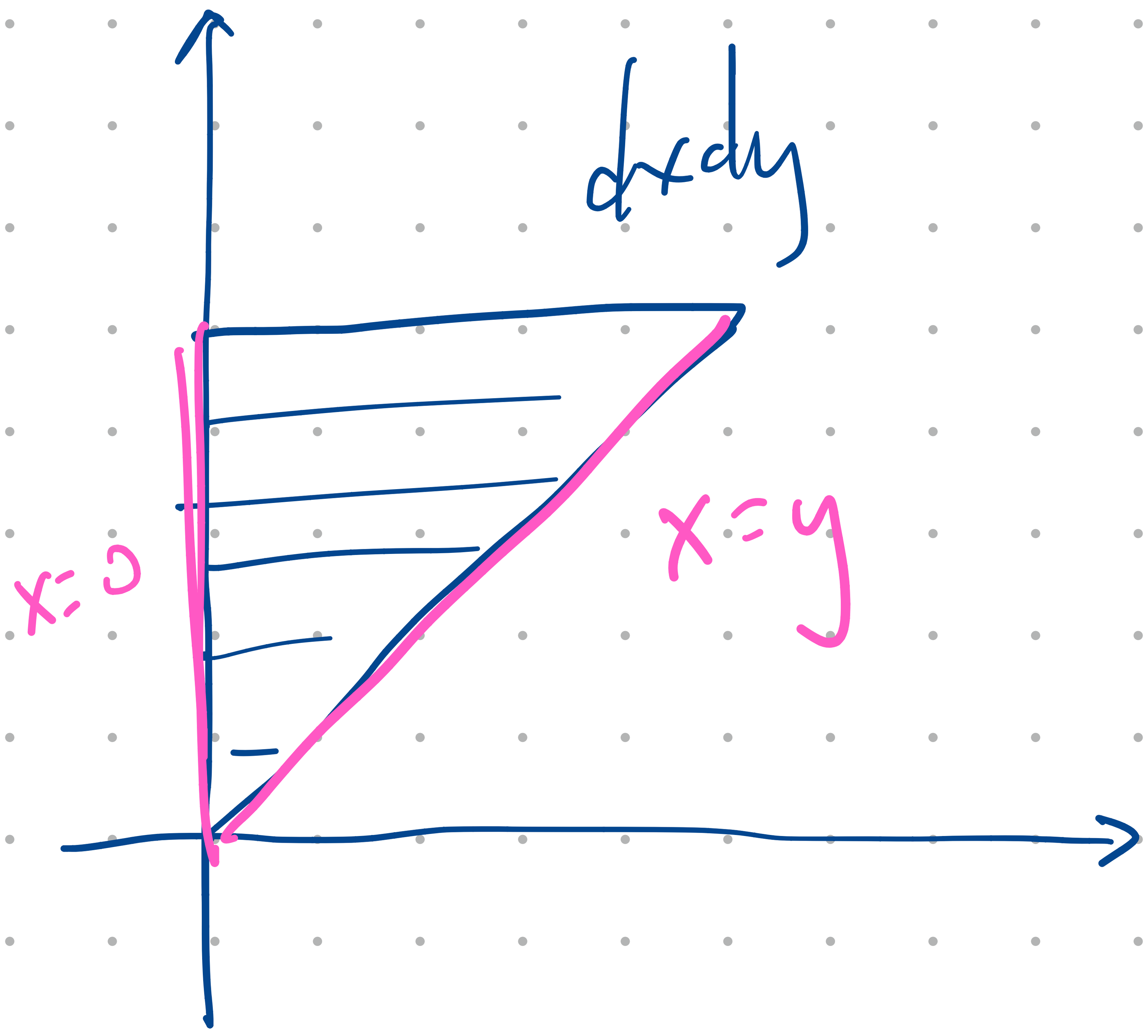
Compute $$\int_0^1 \int_0^1 \overset{f(x,y)}{\boxed{e^{\max(x^2,y^2)}}}\, dy\, dx$$

($\max(x^2,y^2)$ means the larger of the numbers x^2, y^2

i.e. $\max(a,b) = \begin{cases} a & \text{if } a \geq b \\ b & \text{if } b \geq a \end{cases}$)







$$\int_0^1\int_0^x e^{x^2}\,dy\,dx + \int_0^1\int_0^y e^{y^2}\,dx\,dy$$

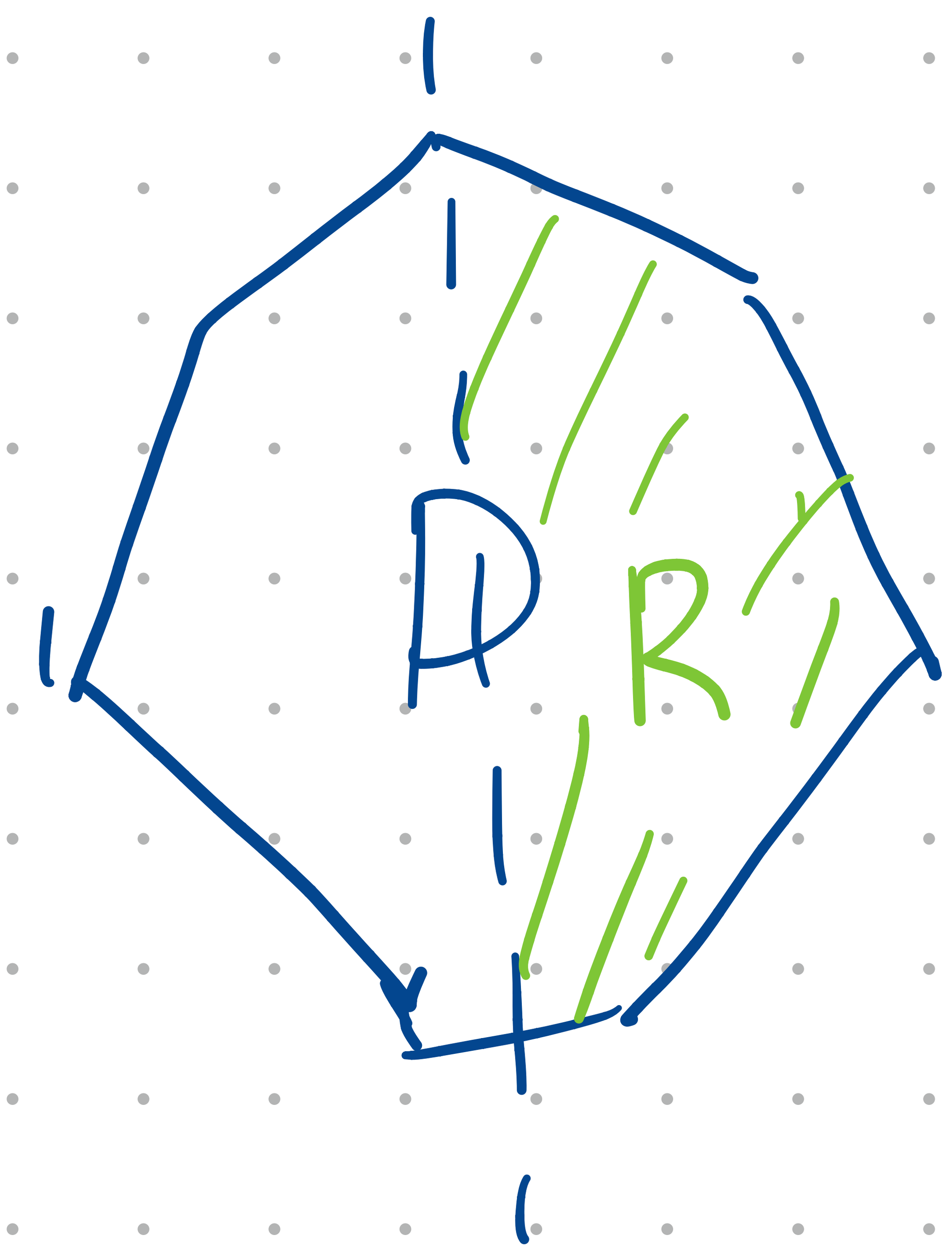
$$= 2\int_0^1\int_0^x e^{x^2}\,dy\,dx$$

$$= 2\int_0^1 \left(ye^{x^2}\right)\Big|_{y=0}^{y=x} dx = 2\int_0^1 xe^{x^2}\,dx$$

$$= \int_0^1 e^u\,du = \boxed{e-1}$$

Beware, in general:

$$\iint_D f(x,y)\,dx\,dy \neq 2\iint_R f(x,y)\,dx\,dy$$



b/c you don't know if f has that kind of symmetry!

Stewart p. 1105 #3)

Find the average value of the function

$$f(x) = \int_x^1 \cos(t^2)\,dt$$

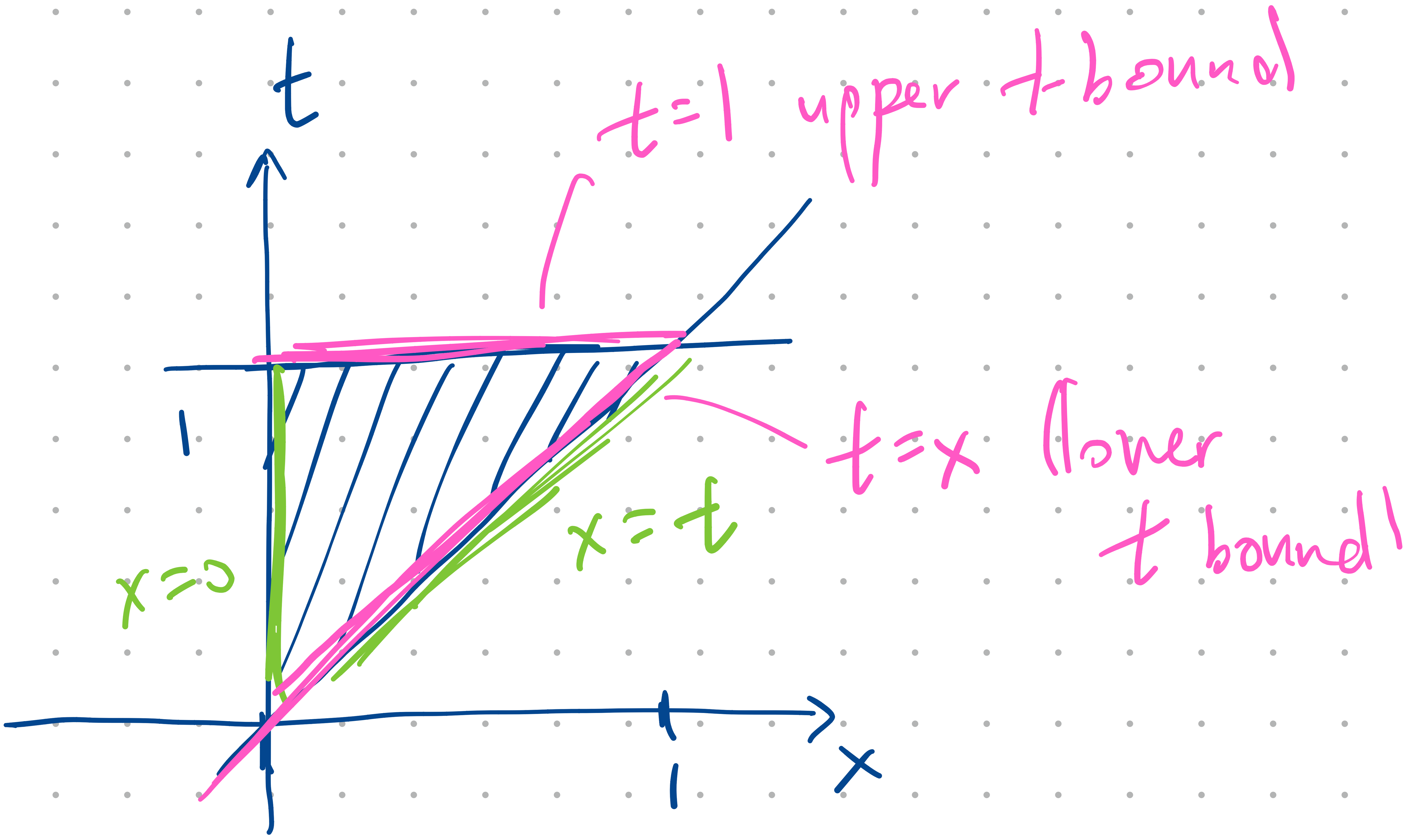
on the interval $[0,1]$.

(The average value of f on an interval $[a,b]$ is given by $\frac{1}{b-a}\int_a^b f(x)\,dx$)

Need to compute

$$\int_0^1 \int_x^1 \cos(t^2)\,dt\,dx$$

$0 \le x \le 1$ $\quad$ $x \le t \le 1$



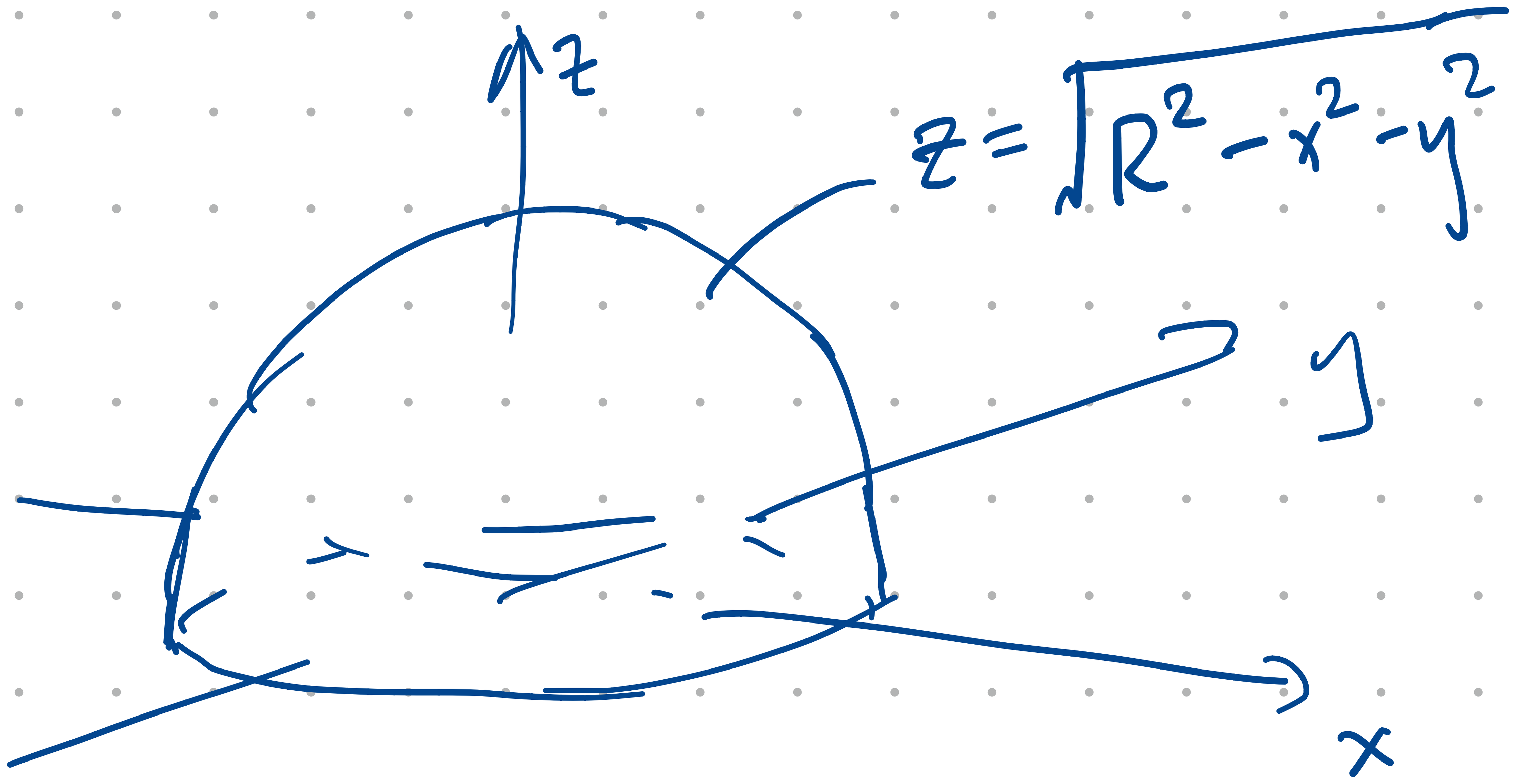
$$\int_0^1 \int_0^t \cos(t^2)\,dx\,dt$$

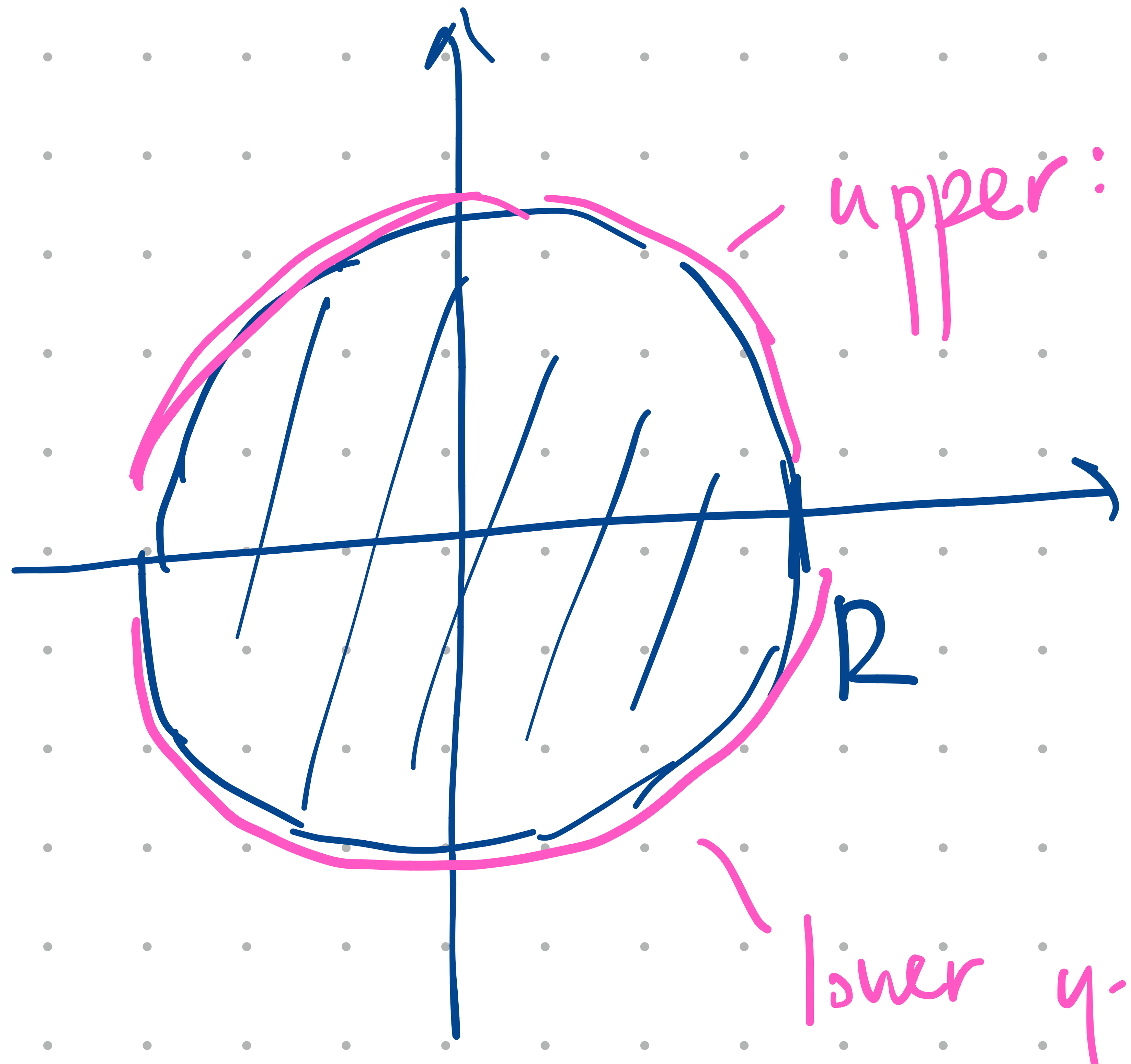
$$= \int_0^1 x\cos(t^2)\Big|_{x=0}^{t}\,dt$$

$$= \int_0^1 t\cos(t^2)\,dt = \dots \text{etc}\dots$$

15.3 ...

Let's say I want the volume of a sphere of radius R. (Ans: $\frac{4}{3}\pi R^3$). Let's do $2 \cdot$ (Vol of hemisphere)





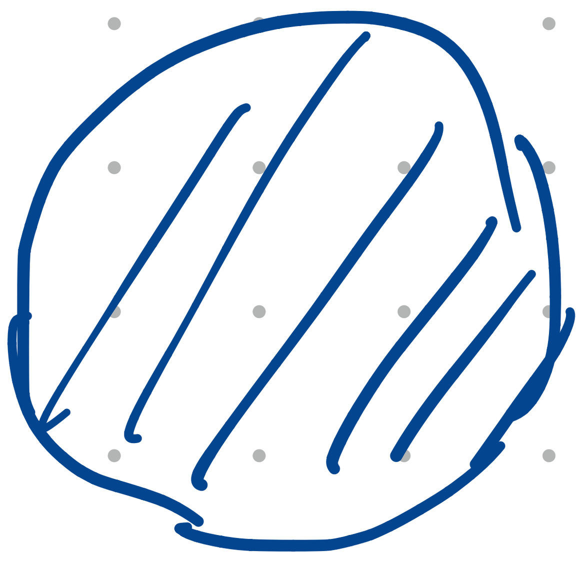
upper: $y = \sqrt{R^2 - x^2}$

lower y-bound

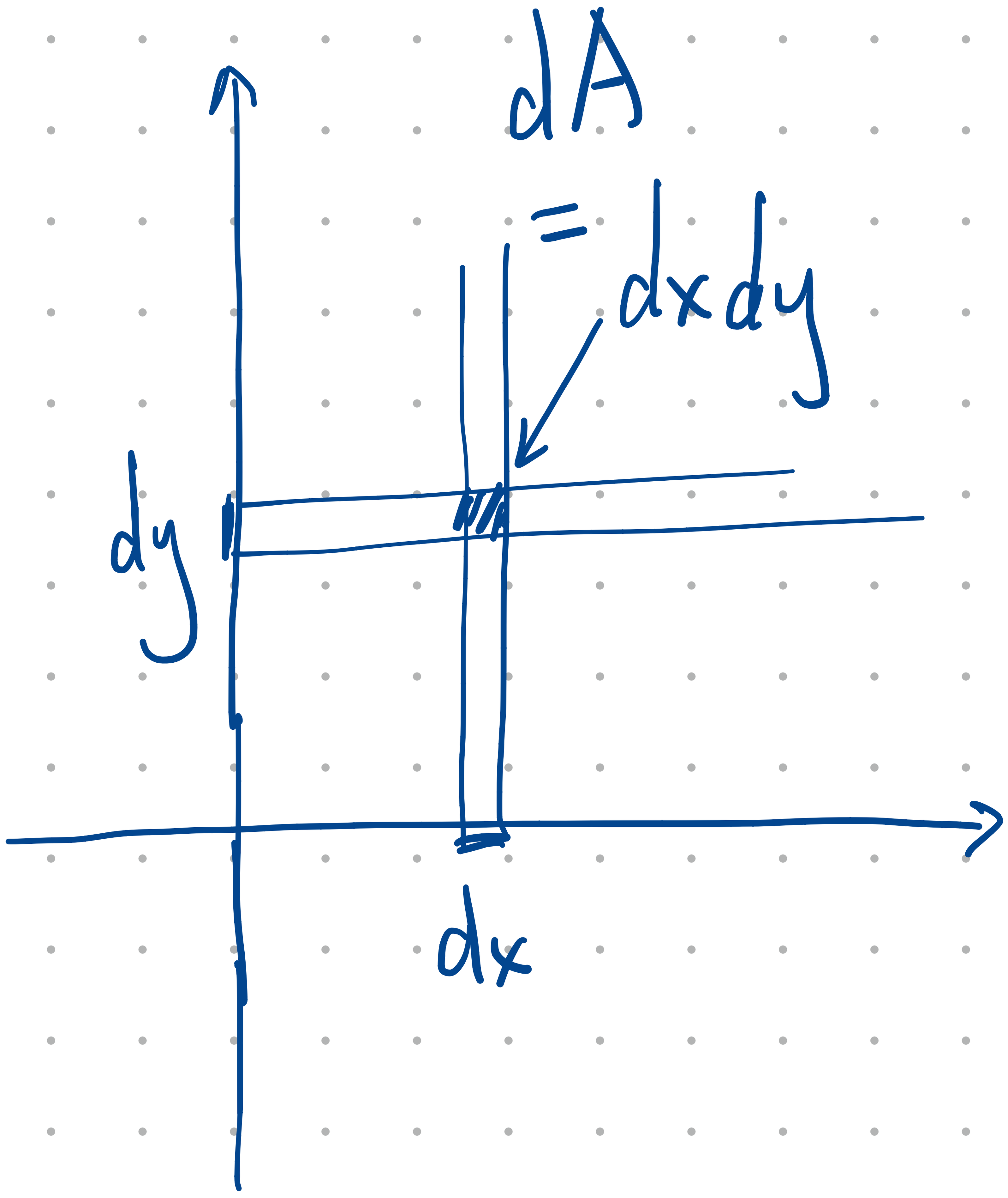
$y = -\sqrt{R^2 - x^2}$

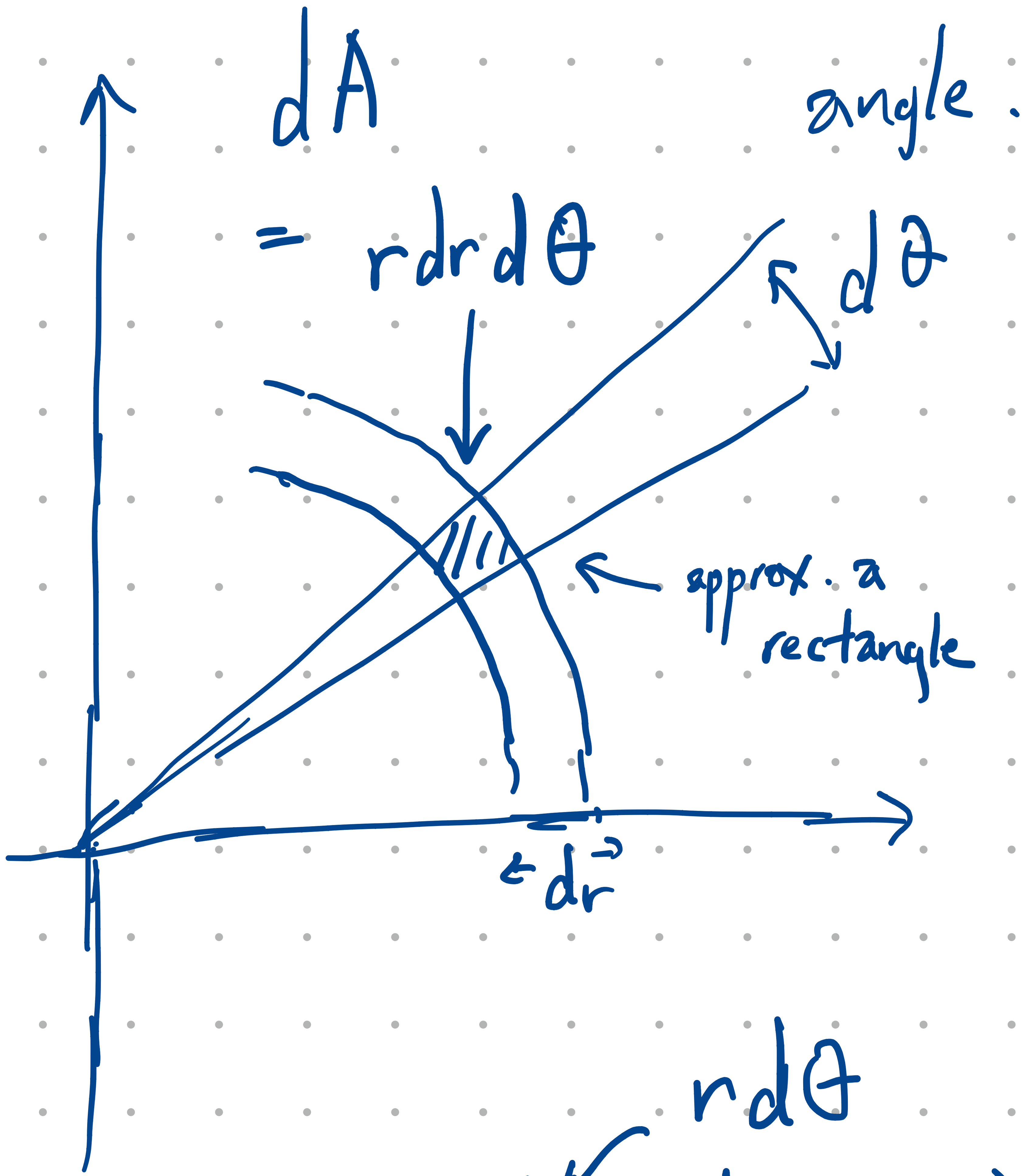
$$\int_{-R}^{R} \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \sqrt{R^2 - x^2 - y^2}\, dy\, dx$$

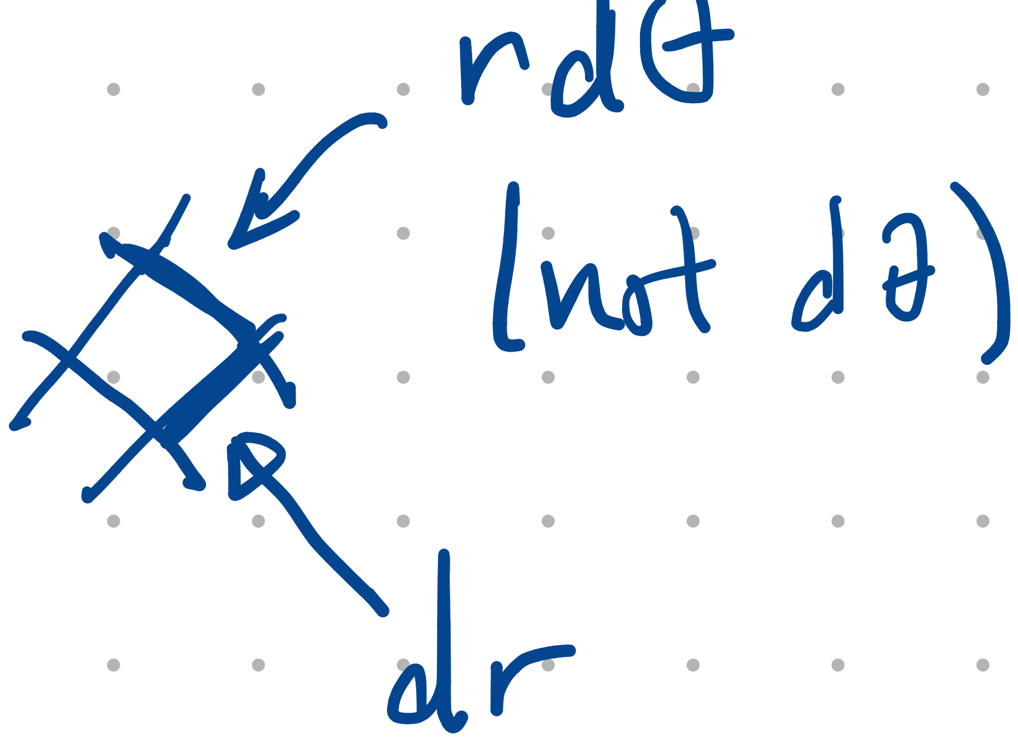
this is not impossible to compute, but it's not great either.



is annoying in rectangular,
but very simple in polar.





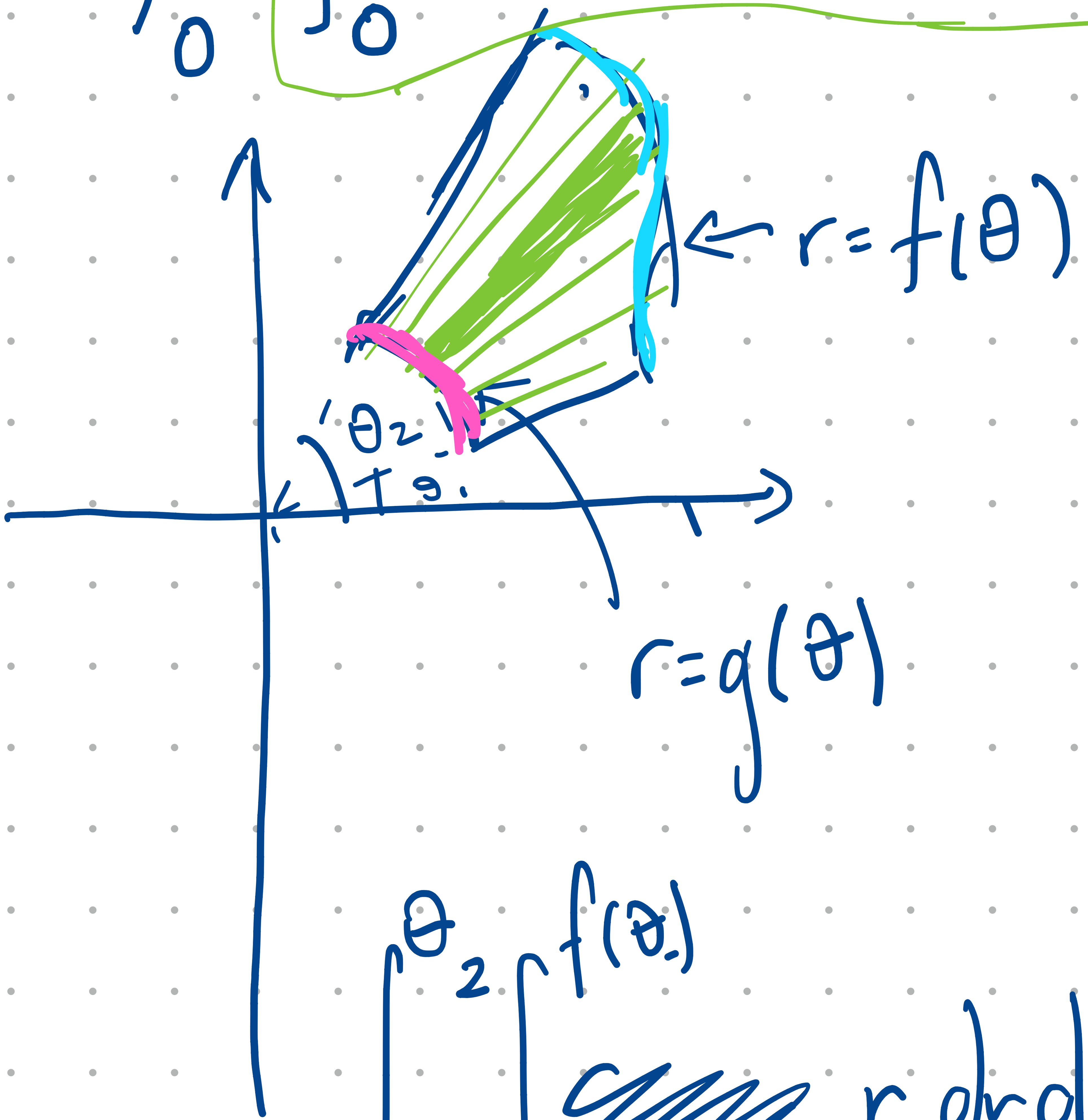


Let's redo the integral in polar:

$x = r\cos\theta$
$y = r\sin\theta$

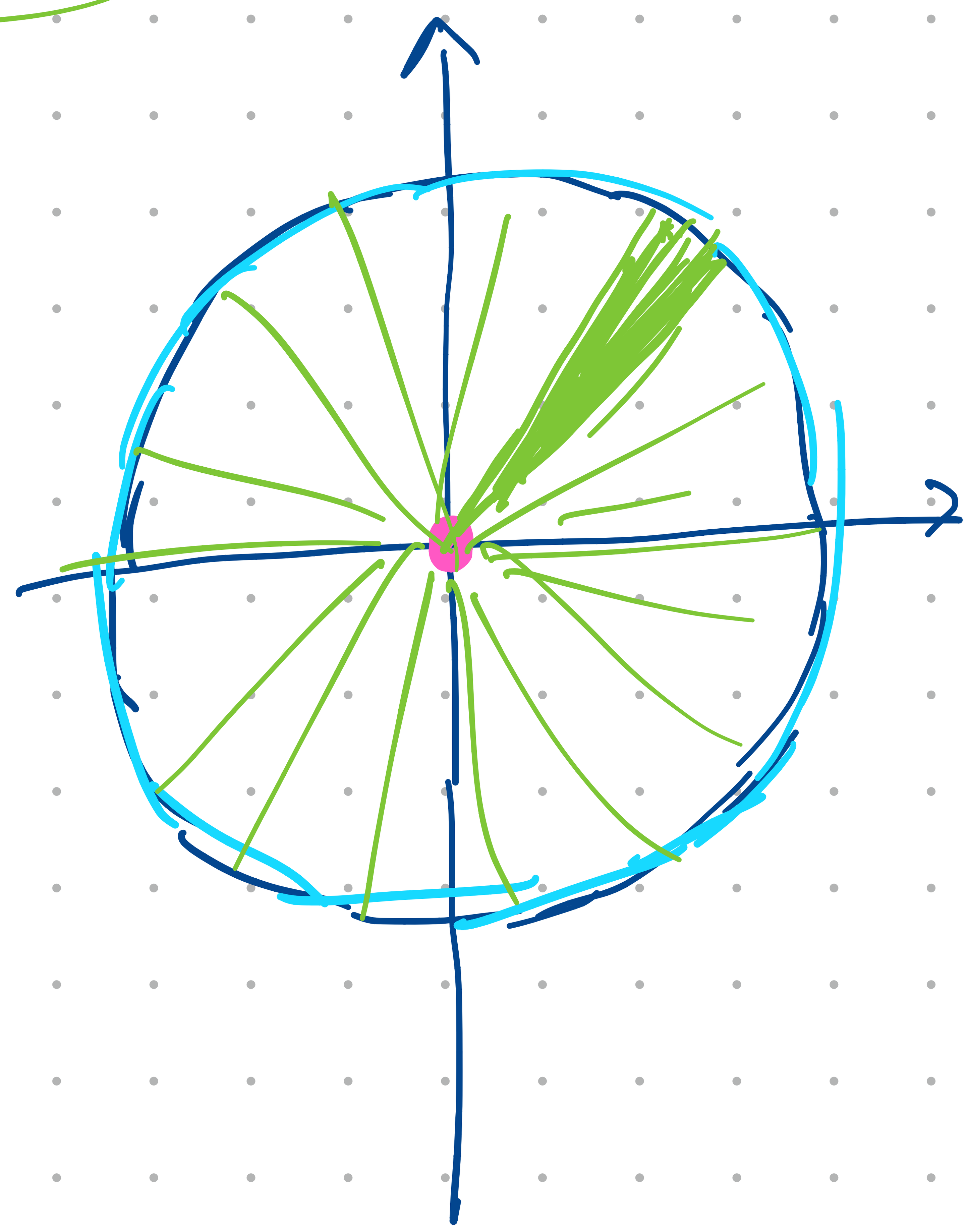
$$z = \sqrt{R^2 - x^2 - y^2} = \sqrt{R^2 - r^2}$$

$$\int_0^{2\pi} \boxed{\int_0^R \sqrt{R^2 - r^2}\; r\,dr\,d\theta}$$



$$\int_{\theta_1}^{\theta_2}\int_{g(\theta)}^{f(\theta)} \quad r\,dr\,d\theta$$

Our situation:



Let's compute the integral: $u = R^2 - r^2$

$du = -2r\,dr$

$$\int_0^{2\pi} \int_{R^2}^{0} \sqrt{u} \cdot \left(-\frac{1}{2}\right) du\, d\theta$$

$$= \int_0^{2\pi} \left(-\frac{1}{3} u^{3/2}\right)\Bigg|_{u=R^2}^{u=0} d\theta$$

$$= \int_0^{2\pi} \frac{1}{3} R^3 \, d\theta = 2\pi \frac{1}{3} R^3$$

So vol of whole sphere is $2 \cdot \left(2\pi \frac{1}{3} R^3\right)$

$$= \boxed{\frac{4}{3}\pi R^3}$$